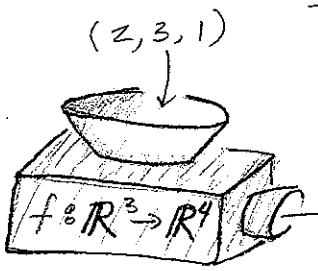
Lecture 8: Functions of several variables (§2.1-2.2)

Last time: Not relevant

HW (Due Tues Feb 5) §2.1: 32, 33
§2.2: Sketch level curves and graphs for 3, 10

Next time: §2.2-2.3

13(a)

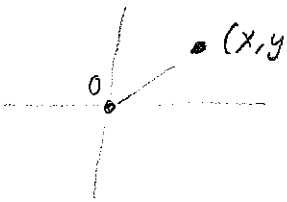


We've already talked about functions of several variables, but only examples were linear transformations.

Let's change that.

Ex: $\mathbb{R}^2 \to \mathbb{R}$

a) $f(x,y)$ = distance from (x,y) to the origin $= \sqrt{x^2+y^2}$



b) $f(x,y)$ = wind speed at (x,y)

c) $W(T,v)$ = effective temperature at actual temp T and wind speed v $= 36 + 0.6T - 36v^{0.16} + 43Tv^{0.16}$

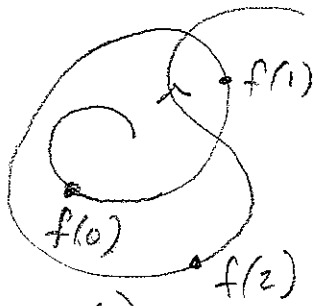
d) $f(x,y) = x^2 - y^2$

e) A 1×2 matrix $A = (a\ b)$ gives a linear trans $\mathbb{R}^2 \to \mathbb{R}$

$$f(x,y) = A\begin{pmatrix} x \\ y \end{pmatrix} = (a\ b)\begin{pmatrix} x \\ y \end{pmatrix} = ax + by$$

something of the form $f(x,y) = ax + by + c$ is called a linear function.

Ex: $\mathbb{R} \to \mathbb{R}^2$ a) lines:



vector valued

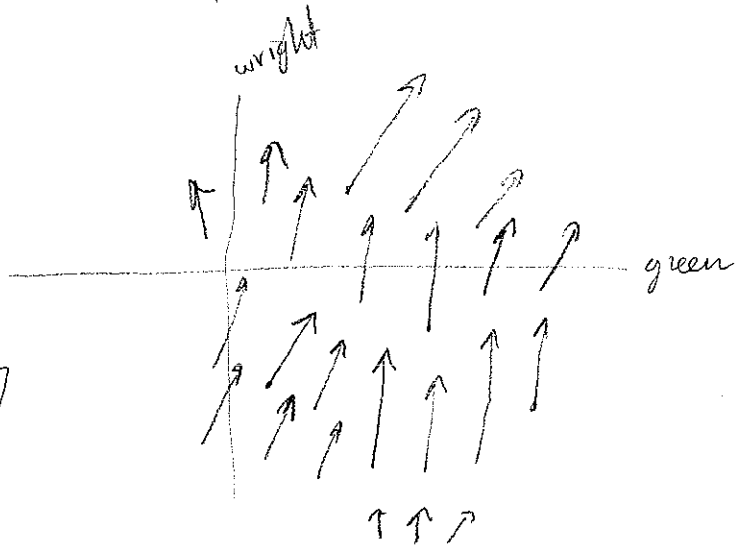
b)

$f(t) = (\text{temp at time } t, \text{ wind speed})$

Ex: $\mathbb{R}^2 \longrightarrow \mathbb{R}^2$ a) Linear transformation

b) Wind speed + direction is a

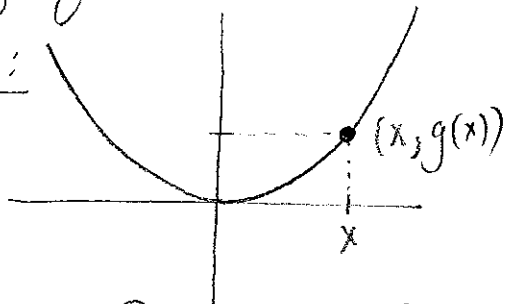
Vector field.



[Other examples: Electric field
Force due to gravity]

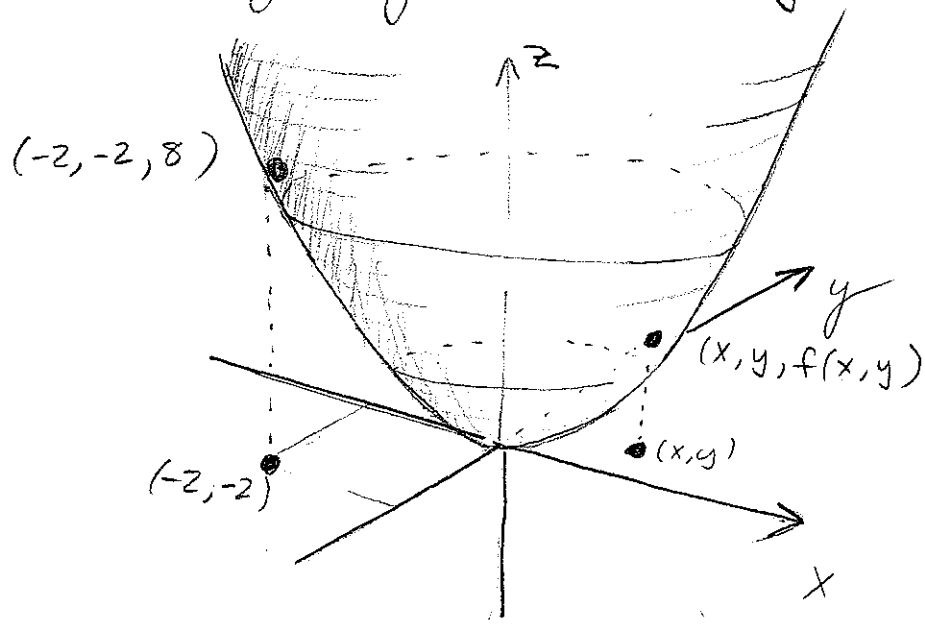
Graphs of functions $\mathbb{R}^2 \to \mathbb{R}$

One var:



$g: \mathbb{R} \to \mathbb{R}$ $g(x) = x^2$

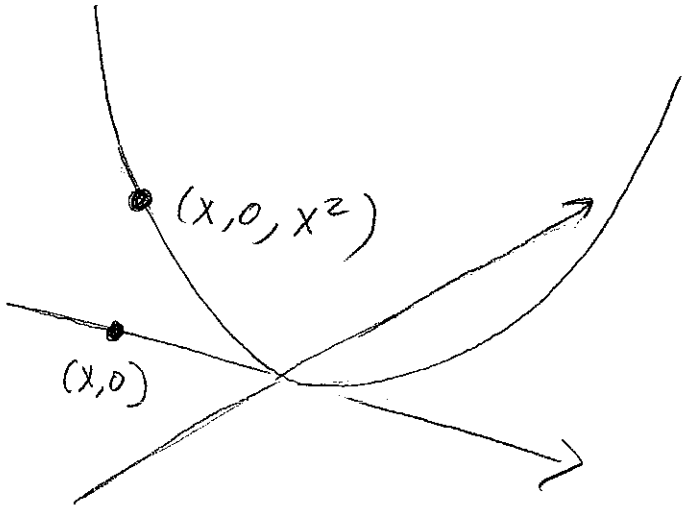
$f: \mathbb{R}^2 \to \mathbb{R}$ given by $f(x,y) = x^2 + y^2$



How to figure out:

Intersect with planes:

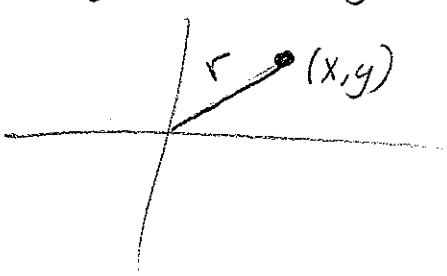
What is over the x-axis?
[or y-axis]



What is the intersection with $\{z = c\}$

i.e. $f(x,y) = c$ or $x^2 + y^2 = c$? A circle of radius $\sqrt{c}$

Symmetry $x^2 + y^2 = r^2$



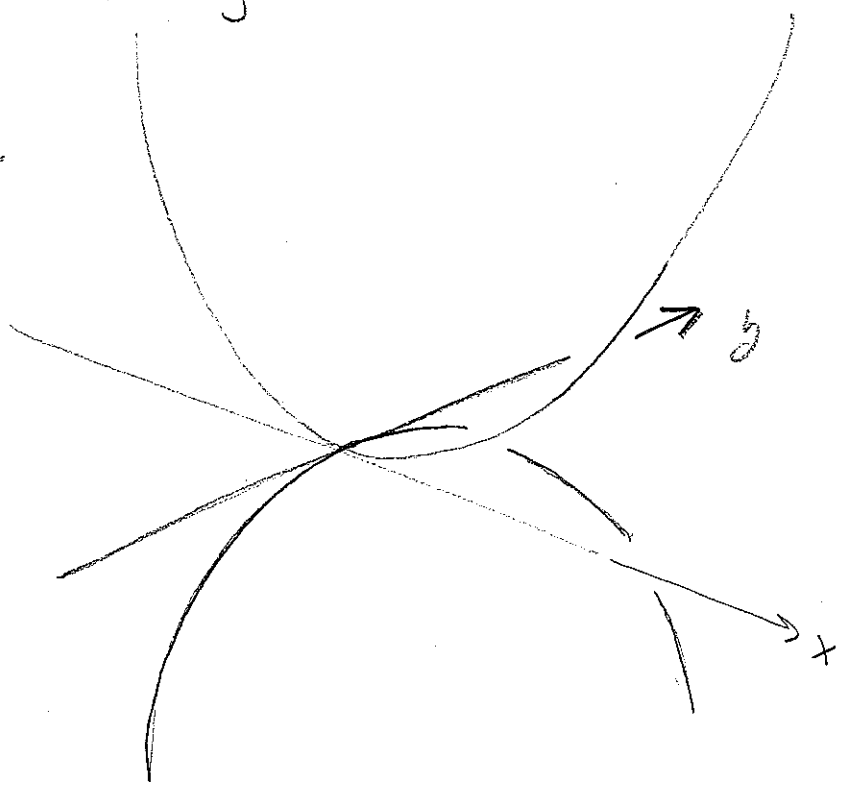
$\Rightarrow$ Graph is invariant under rotation about the z-axis.

Computer: Can be useful

Ex: $f: \mathbb{R}^2 \to \mathbb{R}$ $\quad f(x,y) = x^2 - y^2$

Over x-axis: parabola given by x^2

Over y-axis: parabola given by $-y^2$



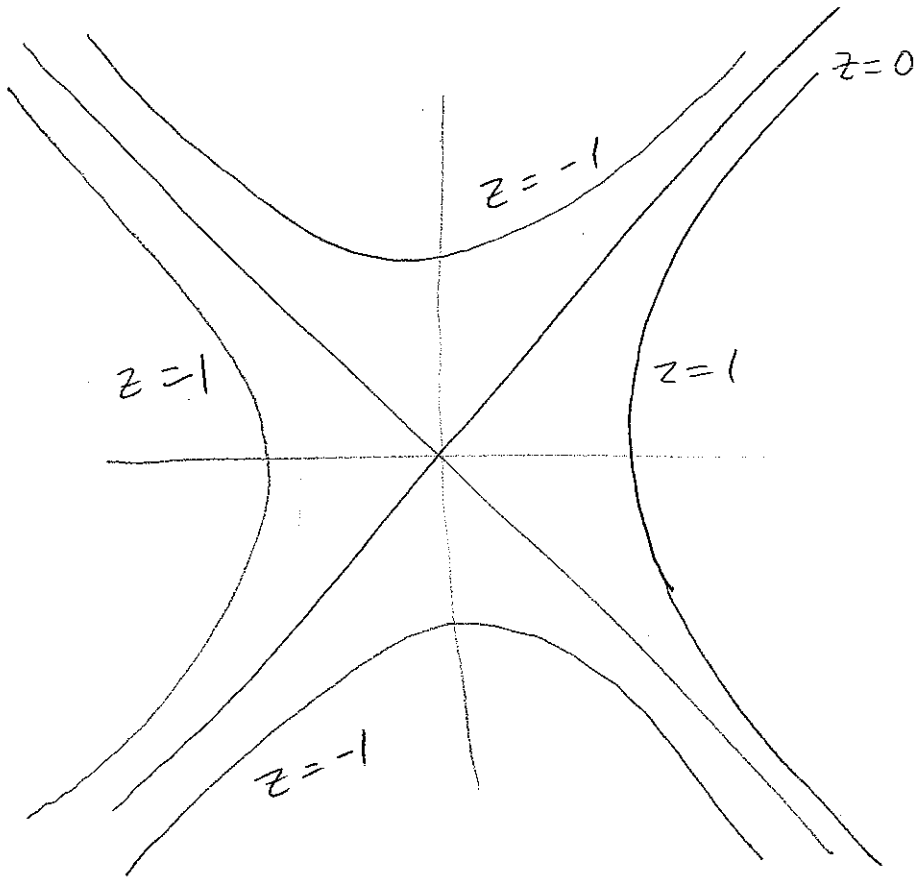
What are the intersections with planes?

$z = 0$: $x^2 - y^2 = 0 \iff x^2 = y^2 \iff x = \pm y$

$z = -1$: $x^2 - y^2 = -1 \iff y^2 = x^2 + 1$

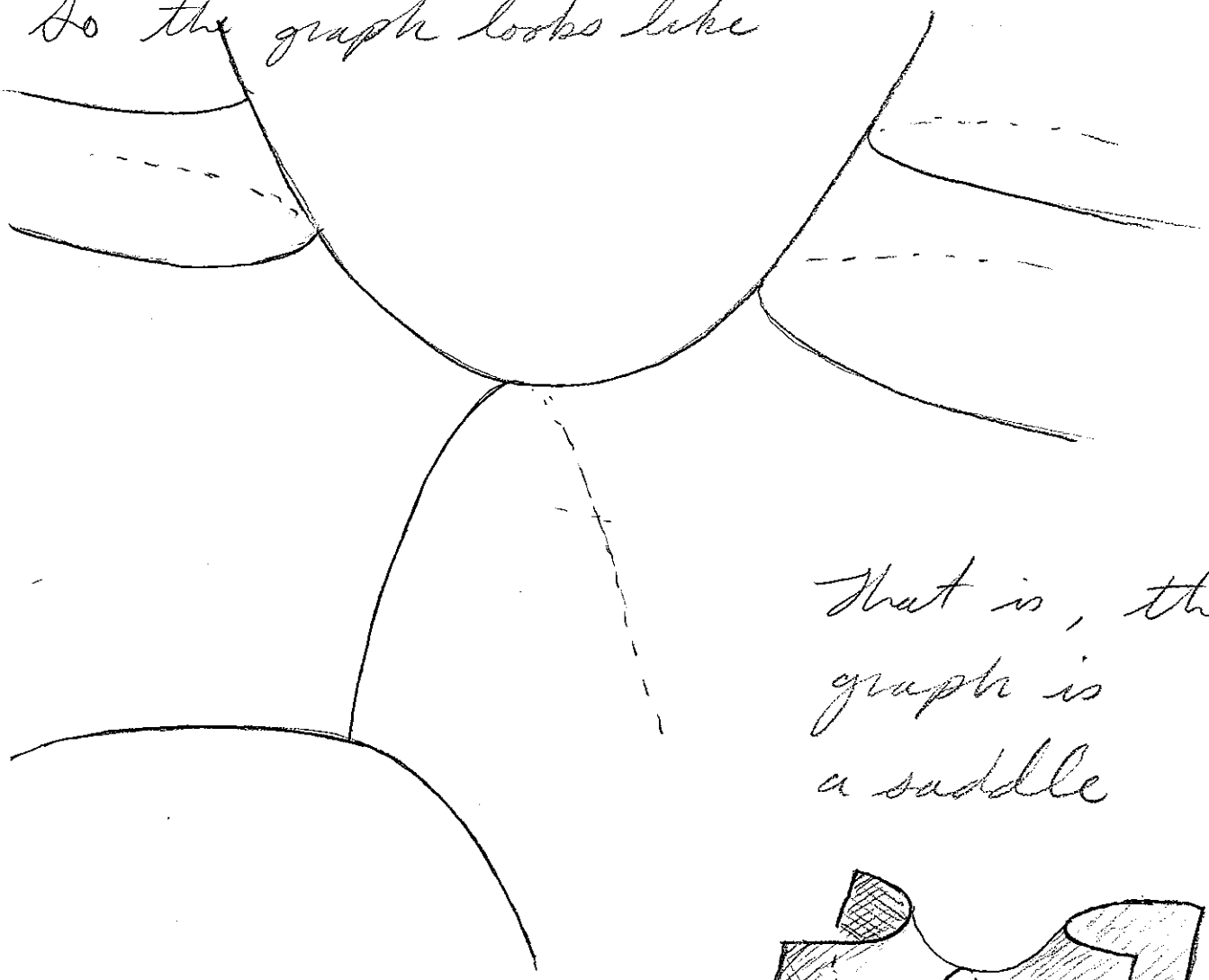
$z = 1$: $x^2 - y^2 = 1$

$\iff x^2 = y^2 + 1$

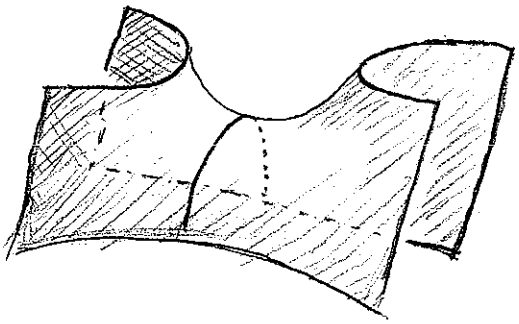


These are called level sets, like contour lines on a map

So the graph looks like



That is, the graph is a saddle



If time remains:

- Do $f(x,y) = 3x - 2y$
- Talk about $f: \mathbb{R}^3 \to \mathbb{R}$:

  Can't see the graph (it's in $\mathbb{R}^4$!) but can still see level sets. Do $x^2+y^2+z^2$.