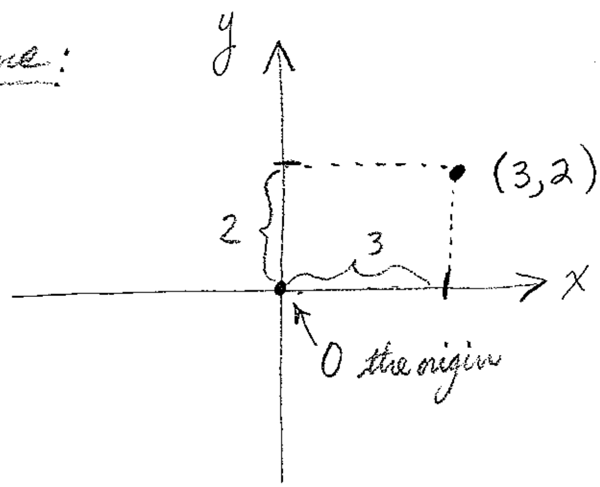
Lecture 2: Today: Vectors in the plane and 3-space

Sections (1.1 – 1.3)

HW: Read Section 1.2
1.1 # 1, 2, 8, 9, 14
1.3 # 6, 10, 11

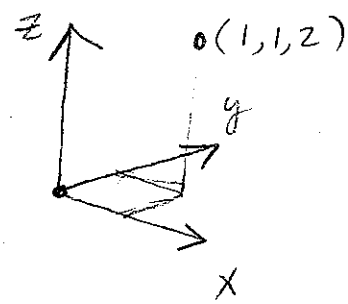
Plane:



Cartesian coordinates keep track of points

$$\mathbb{R}^2 = \left\{ (x,y) \;\middle|\; \begin{array}{l} \text{where } x \text{ and } y \\ \text{are } \underbrace{\text{real \#s}}_{\text{denoted } \mathbb{R}} \end{array} \right\}$$

3-space:



$$\mathbb{R}^3 = \{ (x,y,z) \mid x \overset{\text{in}}{\in} \mathbb{R},\ y \in \mathbb{R},\ z \in \mathbb{R} \}$$

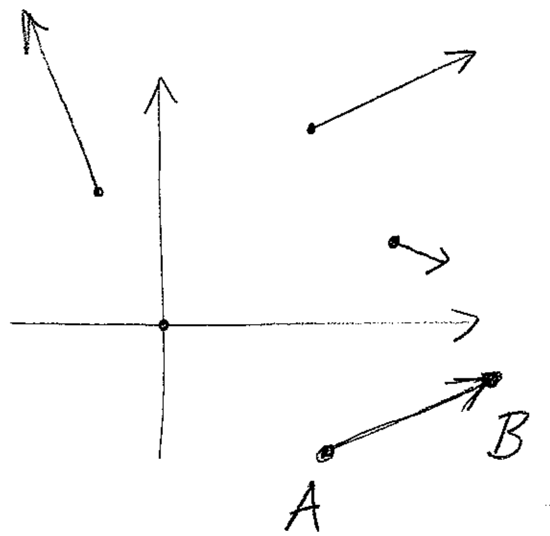
n-dimensional space: $\mathbb{R}^n = \{ \underbrace{(x_1, x_2, \ldots, x_n)}_{\text{ordered tuple}} \mid x_i \in \mathbb{R} \}$

[Point of abstraction in math: to provide intuition about complicated things by looking at analogous simple things. See e.g. multivariable regression.]

The x_i can be distances, times, weights, incomes...

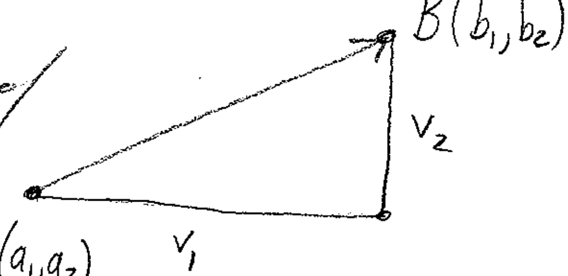
Vectors in the plane:

an arrow where both the direction and the length are important. [used to e.g. keep track of wind speed/direction.]



Keeps track of the relative positions of two points

Keeping track of

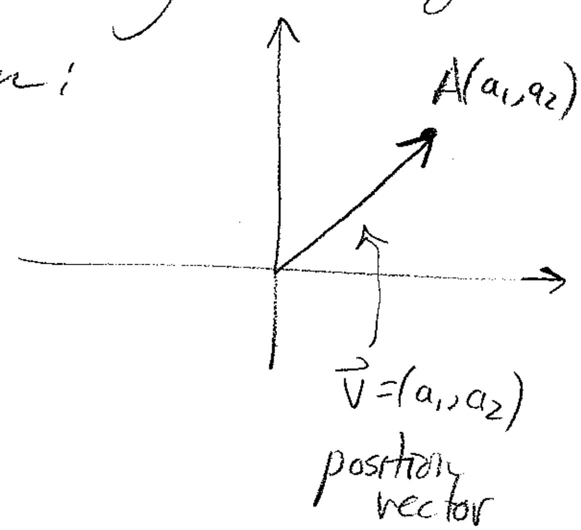


$\vec{V} = (v_1, v_2)$

$= (b_1 - a_1, b_2 - a_2)$

So vectors are also parameterized by $\mathbb{R}^2$, [but they are subtlely different than points.] Going between:

[Often think of vectors as "based" at O]

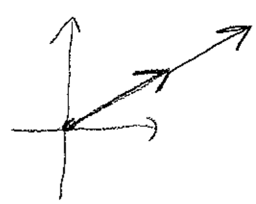


Operations on vectors:

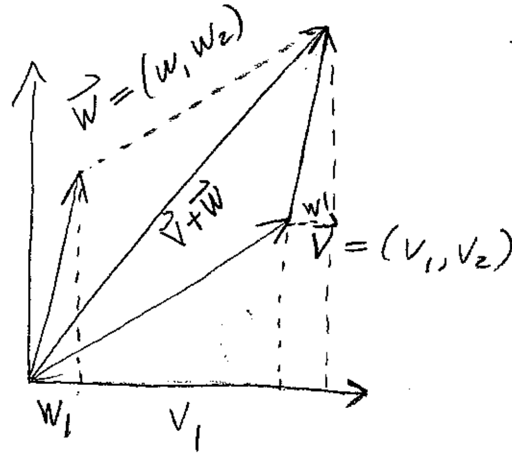
Scaling: $\vec{V} = (v_1, v_2)$

$\alpha \vec{V} = (\alpha v_1, \alpha v_2)$

a number



Addition:



$\vec{V} + \vec{W} = (V_1 + W_1, V_2 + W_2)$

Properties: 1) $\vec{v}+\vec{w}=\vec{w}+\vec{v}$

2) $\vec{u}+(\vec{v}+\vec{w})=(\vec{u}+\vec{v})+\vec{w}$

3) $\alpha(\vec{v}+\vec{w})=\alpha\vec{v}+\alpha\vec{w}$

4) $(\alpha+\beta)\vec{v}=\alpha\vec{v}+\beta\vec{v}$

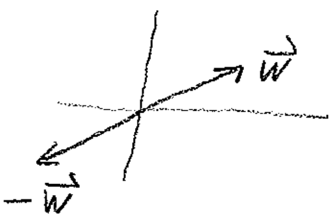
5) $(\alpha\beta)\vec{v}=\alpha(\beta)\vec{v}$

Subtraction: $\vec{v}-\vec{w}$

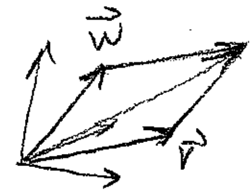
$= \vec{v}+(-\vec{w})$

$\quad\quad\quad \| $

$\quad\quad (-1)\vec{w}$



Same idea for $\mathbb{R}^3$ (and $\mathbb{R}^n$)



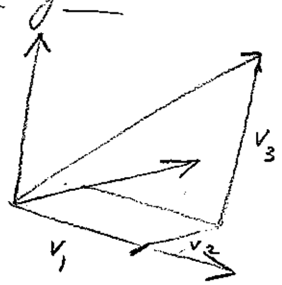
What about multiplication?

[For $\mathbb{R}^3$, there are two kinds, both somewhat different from mult of #s.]

Dot product: $\vec{v}=(v_1,v_2,v_3)$ $\quad \vec{w}=(w_1,w_2,w_3)$

$$\vec{v}\cdot\vec{w}=v_1w_1+v_2w_2+v_3w_3$$

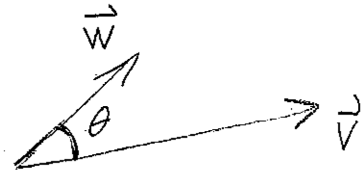
Length:



$$\|v\|=\sqrt{v_1^2+v_2^2+v_3^2}$$

[Also called the magnitude or norm]

Point: $\vec{v}\cdot\vec{w}=\|\vec{v}\|\|\vec{w}\|\cos\theta$



$0\leq\theta\leq\pi$

[N.B. Smaller of two angles ]

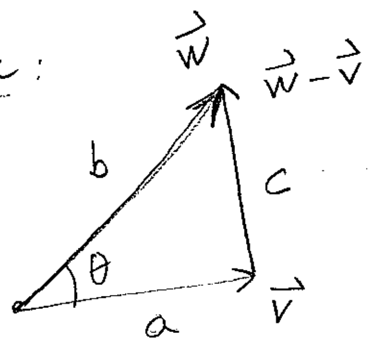
Properties: $\|\vec{v}\|^2 = \vec{v}\cdot\vec{v}$

$$\vec{v}\cdot\vec{w} = \vec{w}\cdot\vec{v}$$

$$\vec{u}\cdot(\vec{v}+\vec{w}) = \vec{u}\cdot\vec{v} + \vec{u}\cdot\vec{w}$$

$$(\alpha\vec{v})\cdot\vec{w} = \alpha(\vec{v}\cdot\vec{w})$$

Consider:



$a = \|\vec{v}\|$ $\quad b = \|\vec{w}\|$

$$c^2 = \|\vec{w}-\vec{v}\|^2 =$$

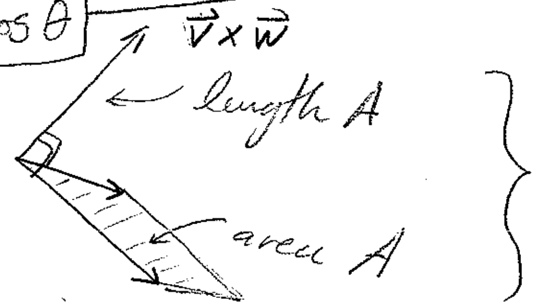
$$(\vec{w}-\vec{v})\cdot(\vec{w}-\vec{v}) =$$

$$\vec{w}\cdot\vec{w} - \vec{v}\cdot\vec{w} - \vec{w}\cdot\vec{v} + \vec{v}\cdot\vec{v} = a^2+b^2-2\boxed{\vec{v}\cdot\vec{w}}$$

$$=$$

Law of Cosines:

$$c^2 = a^2+b^2-2\boxed{ab\cos\theta}$$



Later: Cross product

$$\vec{v}\times\vec{w} = -\vec{w}\times\vec{v}$$