Math 241 F1H: Problem Set 9

Due date: Tuesday, April 8.

Midterm: The third midterm exam will be held in class on Thursday, April 10.

Office Hours: My office hours next week will be

- Monday: 3-5.
- Tuesday: 9–10:30 and 4:00–5:30.
- Wednesday: 9–10:30 and 3:00–5:00.

Review: Wednesday's lecture will be a review session. As always, please send me suggestions for topics.

1. Section 7.3: #16.

2. Section 7.3: #17.

3. Section 7.4: #3, 5, 6.

4. Section 7.4: #15. (See Example 2.90 on page 146 for the setup on heat flow. In this problem, the conductivity is to be taken to be 1.)

5. Section 7.4: #21.

6. Section 7.4: #23.

7. Verify Green's Theorem for the region and vector field given in §8.1 #12, that is, compute the line integral $\int_C \mathbf{F}\cdot ds$ directly and compare the result to $\iint_D \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right) dA$.

8. Let D be a region in $\mathbb{R}^2$ which is star-shaped about $\mathbf{0}$. That is, the boundary of D can be parameterized by
$$c(t) = \big(f(t)\cos t, f(t)\sin t\big) \quad \text{for } 0 \le t \le 2\pi,$$
where $f\colon [0,2\pi] \to [0,\infty)$ satisfies $f(0) = f(2\pi)$ so that this gives us a closed curve.

   Without using Green's Theorem, show that for the vector field $\mathbf{F} = \frac{1}{2}(-y, x)$ one has
$$\int_{\partial D} \mathbf{F}\cdot ds = \text{Area(D)}$$
by using the parameterization above.

9. Section 8.2: #2.

10. Section 8.2: #7.

11. Section 8.2: #13.

12. Consider a region D in $\mathbb{R}^2$ with a single boundary component C, and let $\mathbf{n}$ be the outward-pointing unit vector field.

    Given a vector field $\mathbf{F} = (F_1, F_2)$ on D, define a new vector field by $\mathbf{G} = (-F_2, F_1)$. Show that

(a) $\int_C \mathbf{F} \cdot \mathbf{n}\, ds = \int_C \mathbf{G} \cdot ds$

(b) $\iint_D div\, \mathbf{F}\, dA = \iint_D \left(\frac{\partial G_2}{\partial x} - \frac{\partial G_1}{\partial y} \right) dA.$

Explain why this means that Green's Theorem and the Divergence Theorem in $\mathbb{R}^2$ are equivalent.

Note: This assignment is complete.