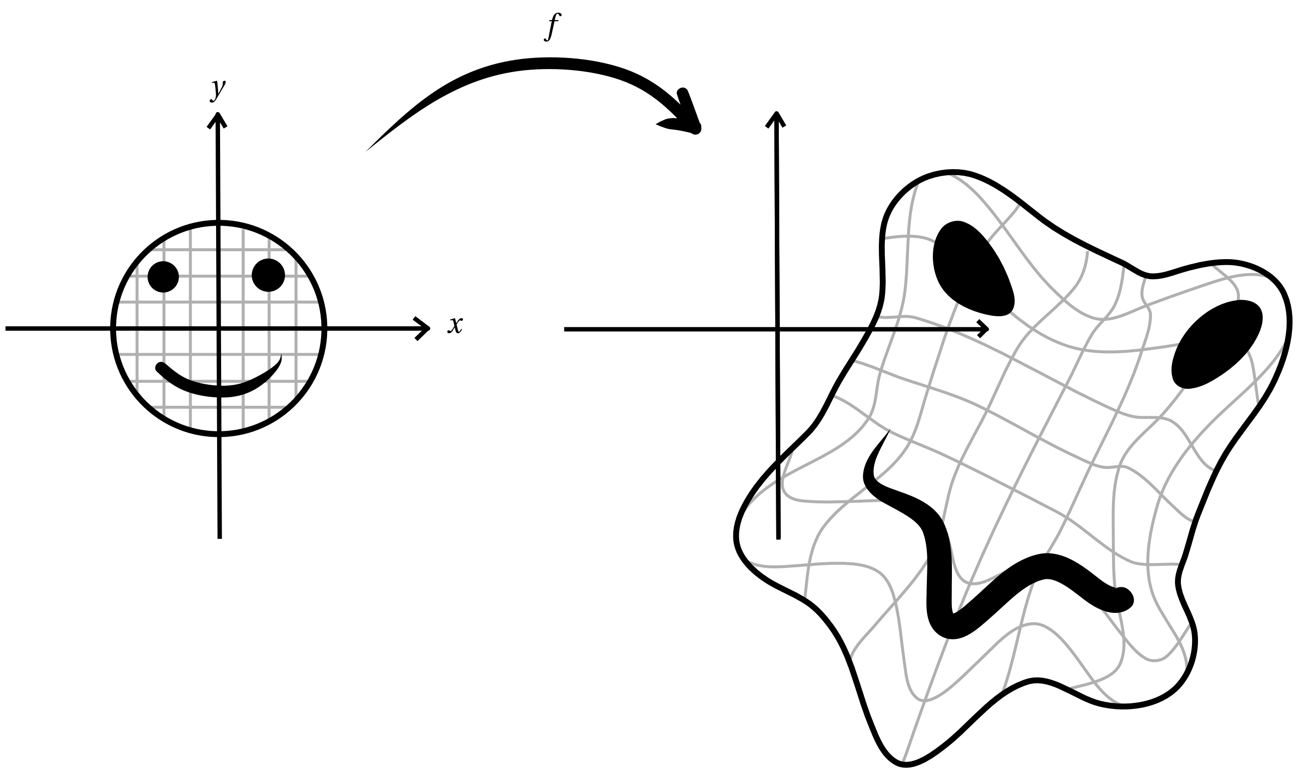
Math 241 F1H: Problem Set 5

Due date: In class on Tuesday, February 26.

1. Redo Problem 5 from the exam: Consider the function $f\colon \mathbb{R}^2 \to \mathbb{R}^2$ which distorts the plane as show below:



Compute $Df(0,0)$, taking it as given that the entries of matrix are just integers (no roots, fractions, trig functions, etc., just integers). Hint: The determinant of your answer should be negative — look at the corners of the smile in the image.

2. §4.1: #1–4.

3. §4.2: #30. Hint: The error of these approximations is $< 10^{-3}$ in the linear case and $< 10^{-4}$ in the quadratic case.

4. §4.2: #31.

5. The problem will help you understand where the second derivative comes from. Consider the function $f\colon \mathbb{R}^2 \to \mathbb{R}$ given by $f(x, y) = x^2 + xy + y^2$.

   (a) Check that f has a critical point at $\mathbf{0}$.

(b) As we've talked about, given vectors $\mathbf{c}_1, \mathbf{c}_2$ we get new coordinates on $\mathbb{R}^2$ by

$$(u, v) \leftrightarrow u\mathbf{c}_1 + v\mathbf{c}_2$$

Find an *orthonormal* pair of such vectors so that

$$f(u, v) = au^2 + bv^2.$$

(c) Based on your answer in part (b), determine whether f has a local min, max, or saddle at $\mathbf{0}$.

(d) Compute the Hessian matrix of f with respect to both the (x, y) and (u, v) coordinates. Check both matrices have the same determinant.

In general, you can always find such coordinates and the determinant of the Hessian remains unchanged, and this is why the second derivative test works. In any dimension finding such coordinates is related to finding eigenvalues/eigenvectors of the Hessian matrix...

6. §4.3: #3–6.
7. §4.3: #11.
8. §4.3: #20.
9. §4.3: #31.
10. §4.4: #3–6.
11. §4.4: #13.
12. Redo problem §4.3 #20 using Lagrange multipliers.

Note: This assignment is complete.