Math 241 F1H: Last Problem Set (#11)

Due date: Tuesday, April 29.

1. Section 8.4: #1,3,5,7,9. Note: I didn't mention this in class, but it's considered improper to add forms of different degrees; e.g. $dx + dy \wedge dz$ is not allowed.
2. Section 8.4: #13.
3. Section 8.4: #23.
4. Section 8.4: #26.
5. Section 8.4: #28.
6. Section 8.4: #29.
7. Section 8.4: #34.
8. Section 8.4: #37.
9. For a vector field $\mathbf{F} = (F_1, F_2, F_3)$ on $\mathbb{R}^3$, there is a corresponding 1-form
$$\alpha = F_1 dx + F_2 dy + F_3 dz$$
and also a corresponding 2-form
$$\beta = F_1\ dy \wedge dz + F_2\ dz \wedge dx + F_3\ dx \wedge dy$$
In this problem, you'll check two computations that I used in class to interpret Stokes' Theorem in the context of differential forms.
   (a) Suppose S is a surface in $\mathbb{R}^3$ parameterized by $r\colon\ D \to S$. Show that for each (u, v) in D one has
$$\mathbf{F}\big(r(u,v)\big) \cdot (T_u \times T_v) = \beta_{r(u,v)}(T_u, T_v)$$
   Conclude that $\iint_S (\mathbf{F} \cdot \mathbf{n})\, dA = \int_S \beta$.
   (b) For the 1-form α, show that $d\alpha$ corresponds to curl $\mathbf{F}$.
10. Consider a 2-form α on a surface S in $\mathbb{R}^3$. Suppose S is closed, that is, it has no boundary.
   (a) If α is exact, i.e. $\alpha = d\beta$ for some 1-form β, what can you say about $\int_S \alpha$? Explain your reasoning.
   (b) If α is closed, i.e. $d\alpha = 0$, does your conclusion in part (a) still hold? **Note:** Here α may only be defined on S, so be careful trying to apply Stokes' Theorem.

Note: This assignment is complete.

Final Exam: The final exam will be Monday, May 5 from 1:30-4:30 in our usual classroom. It will be a comprehensive exam, though material covered since the third midterm will receive special prominence. As always, the general verticality of mathematics tends to lead to an emphasis on later material, at least at the surface level (e.g. you can't do a surface integral without reducing it to a double integral, or differentiate a 2-form without knowing what a partial derivate is). To allow for the wider range of material, you will be allowed to bring up to *three* sheets of notes to the exam. Additional office hours, etc. will be announced shortly.