Lecture 9:

HW #4: §6.4 # 13, 31, 37.

Midterm Friday: usual place / time
Bring one sheet of notes

Wednesday review: email me questions

Review exercises: 1–50, 61–68, 70, 71

Partial Fractions:

$$\frac{P(x) \leftarrow \deg < n}{\underbrace{(a_1x+b_1)\cdots(a_nx+b_n)}_{\text{no repeat factors}}} = \frac{c_1}{a_1x+b_1} + \frac{c_2}{a_2x+b_2} + \cdots + \frac{c_n}{a_nx+b_n}$$

Variations:

Repeat factors: $\frac{x-2}{(x+1)^2}$

Method doesn't work:
$$\frac{x-2}{(x+1)^2} = \frac{A}{x+1} + \frac{B}{x+1} = \frac{A+B}{x+1}$$

Fix:
$$\frac{x-2}{(x+1)^2} = \frac{A}{x+1} + \frac{B}{(x+1)^2} = \frac{A(x+1)+B}{(x+1)^2}$$

Leads to $A = 1$, $B = -3$

$$= \frac{1}{x+1} - \frac{3}{(x+1)^2}$$

General form: $\dfrac{P(x)}{(ax+b)^n} = \dfrac{c_1}{ax+b} + \dfrac{c_2}{(ax+b)^2} + \cdots + \dfrac{c_n}{(ax+b)^n}$ $\quad P(x) \sim \deg < n$

[Can be mixed and matched with other linear factors.]

Higher degree numerator: Why this is a problem:

$$\frac{A}{x+1} + \frac{B}{x-1} = \frac{(A+B)x + (A-B)}{(x+1)(x-1)}$$

Solution: divide numerator first (see Ex 4.3 in the text.)

Higher order factors in the denominator, e.g. x^2+1.

[Summary of integration techniques; things that can't be integrated in simple terms.]

Elementary Terms:

Built from x, constants, e, $\ln$, trig functions (including inverses) using $(+, -, \times, \div)$ and composition

Eg.

$$\frac{(x^2-x)}{\ln(x+6)} \tan^{-1}\left(e^{(x+\sin x)^{1/6}}\right)$$

Liouville (1830s) showed

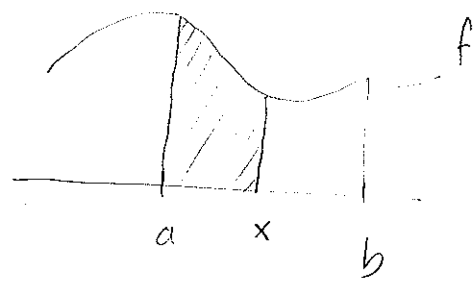
$$e^{-x^2}, \quad \frac{1}{\ln x}, \quad \frac{\sin x}{x}, \quad \frac{e^x}{x}, \quad \ln(\ln(x)),$$

are not the derivative of <u>any</u> expression in <u>elementary terms.</u> However,

<u>Thm:</u> f a continuous function on $[a,b]$. Then

$$F(x) = \int_a^x f(t)\,dt \quad \text{satisfies} \quad F'(x) = f(x) \quad \text{for } x \text{ in } [a,b]$$

<u>Pf:</u> $\displaystyle \lim_{h \to 0} \frac{F(x+h) - F(x)}{h} = \lim_{h \to 0} \frac{1}{h} \int_x^{x+h} f(t)\,dt$



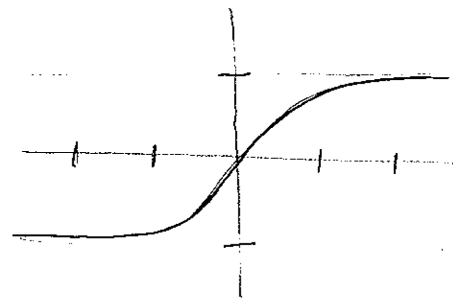
$$\lim_{h \to 0} \begin{pmatrix} \text{average of} \\ f \text{ on } [x, x+h] \end{pmatrix} = f(x).$$

<u>Risch Algorithm:</u> Given an expression in elementary terms, find the antiderivative in elem terms, <u>if this is possible.</u>

Partially implemented in Computer Algebra Systems (Mathematica, Maple, Axiom, etc.) See Section 6.5.

Sometimes, still need to work with the antiderivative

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt$$

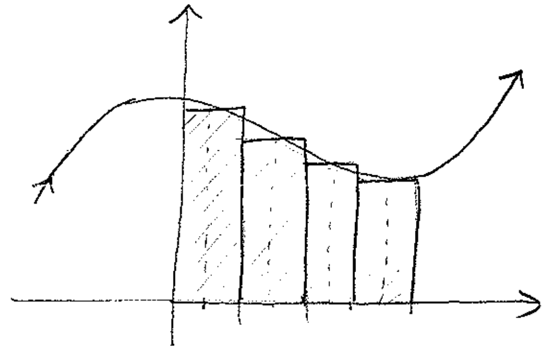


which is used all the time in probability:

Suppose McCain leads Obama by 51 − 49 and $\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$ normal dist
we take a poll with MOE 3.5%. What is the prob that Obama is ahead in the result of the poll?

<u>A</u>: $\frac{1}{2}\left(1+\operatorname{erf}\left(-\frac{1}{2.5}\right)\right) \approx 28.5\%$ $\operatorname{erf}^{-1}(0.95) \approx 1.386$

But how do we compute erf?



- Rieman sums and refinements (Simpsons rule.)

- Keeping track of the amount of error. (Numerical Analysis)

Talk about the exam. // $x^5 - 10x + 2$ S_5 Galois group