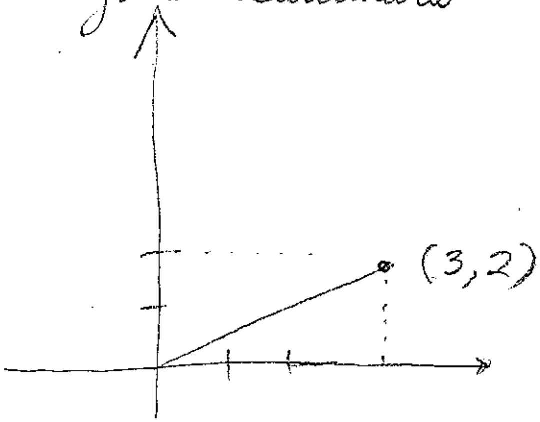
Lecture 32: Polar Coordinates (§9.4)

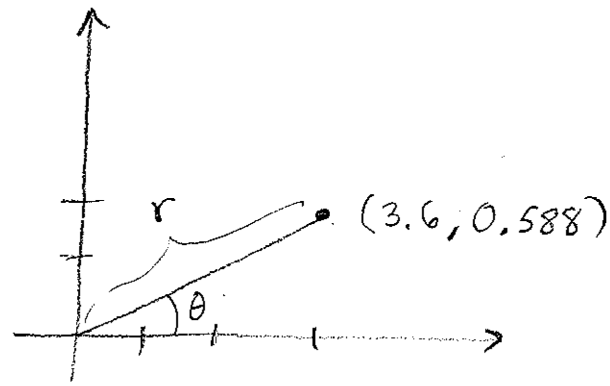
HW (Dec 3): §9.4: 11, 13, 19, 23, 29, 38, 48, 62

Next time: §9.5

Rectangular Coordinates



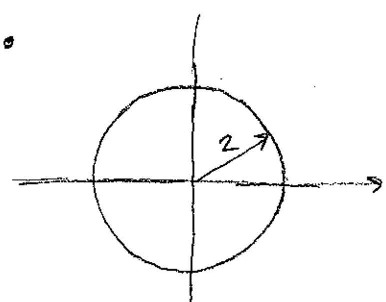
Polar Coordinates



$r = \sqrt{3^2+2^2} = \sqrt{13} \approx 3.6$

$\theta = \tan^{-1}\left(\frac{2}{3}\right) \approx 0.588$

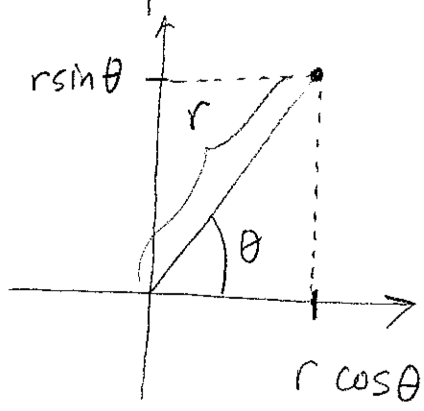
Why? Things like circles are much easier to understand in polar coordinates.



Rect: $x^2+y^2=4$

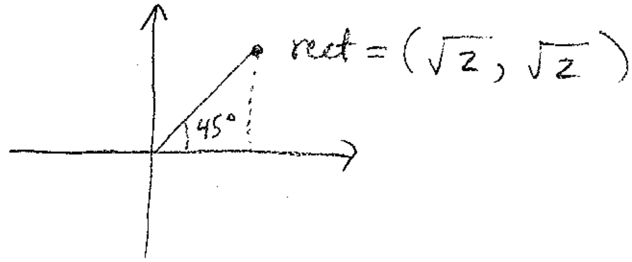
Polar: $r=2$

Converting Polar to Rect:



$(r,\theta) \longrightarrow (r\cos\theta, r\sin\theta)$

Ex: $(r,\theta) = (2, \pi/4)$



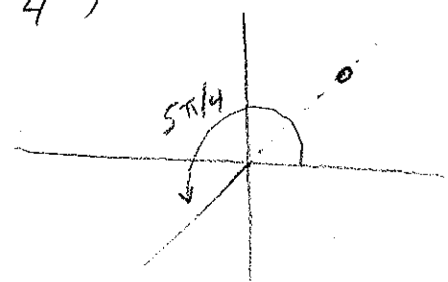
$x = r\cos\theta = 2 \cdot \frac{1}{\sqrt{2}} = \sqrt{2}$

$y = r\sin\theta = 2 \cdot \frac{1}{\sqrt{2}} = \sqrt{2}$

Note: There are multiple polar coord for each point in the plane: $(r,\theta) = (2, \frac{9\pi}{4})$ also gives $(\sqrt{2}, \sqrt{2})$, as does $(r,\theta) = (2, -\frac{7\pi}{4})$ and $(-2, \frac{5\pi}{4})$

$x = -2\cos(5\pi/4) = -2\left(-\frac{1}{\sqrt{2}}\right) = \sqrt{2}$

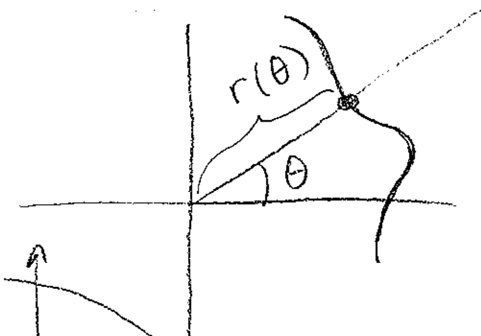
$y = -2\sin(5\pi/4) = -2\left(\frac{-1}{\sqrt{2}}\right) = \sqrt{2}$



Typically: Favor $r > 0$ and θ in $[0, 2\pi)$ (or $[-\pi, \pi)$)

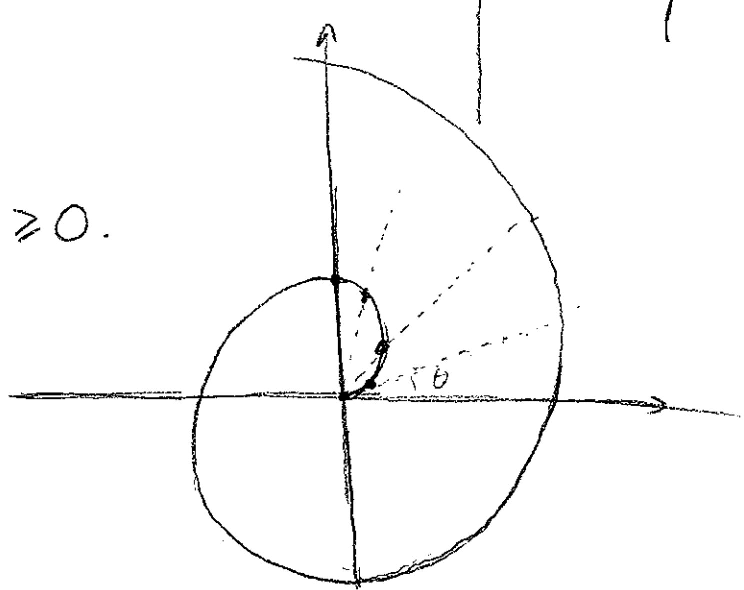
Curves in polar coordinates:

r is a function of θ

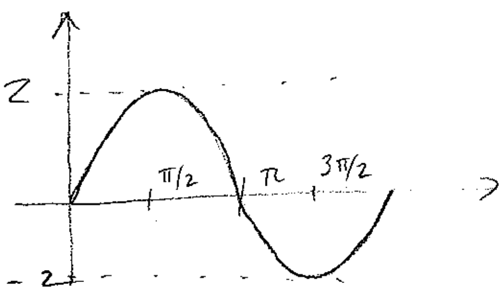


Ex: $r = \theta$ and $\theta \geq 0$.

Archimedian Spiral.

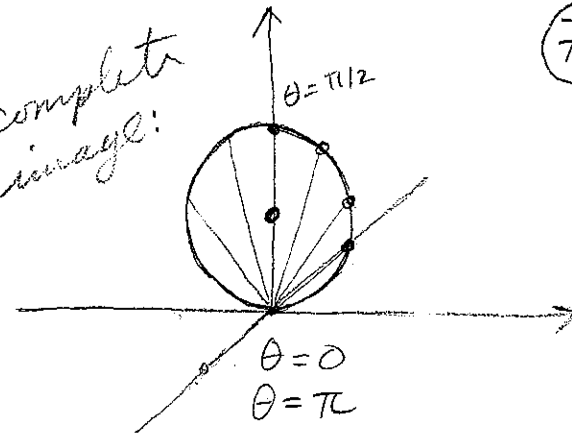


Ex: $r = 2\sin\theta$



$y = 2\sin x$

complete image:



Traces out a circle: $(y-1)^2 + x^2 = 1$

Polar coor: $(\sin\theta, \theta)$

Rect coor: $(r\cos\theta, r\sin\theta) = (2\sin\theta\cos\theta, 2\sin^2\theta)$

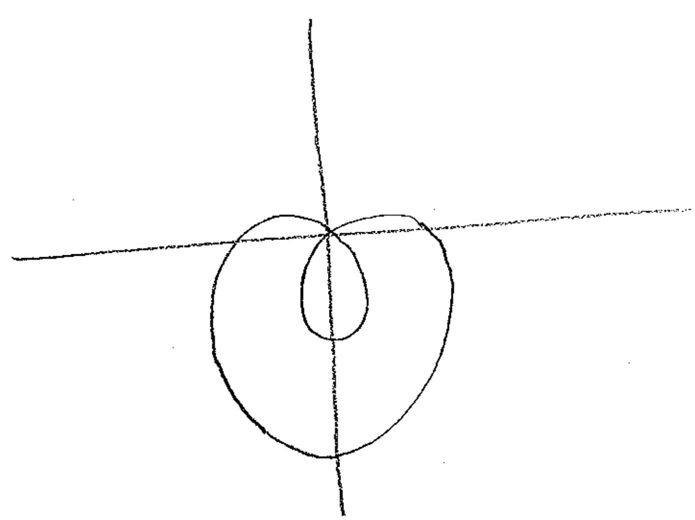
Check: $(y-1)^2 + x^2 = (2\sin^2\theta - 1)^2 + 4\sin^2\theta\cos^2\theta$

$$= 4\sin^4\theta - 4\sin^2\theta + 1 + 4\sin^2\theta\cos^2\theta$$

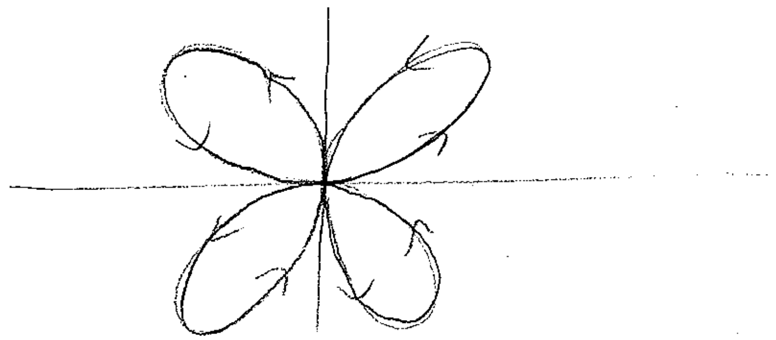
$$= 4\sin^2\theta(\sin^2\theta + \cos^2\theta - 1) + 1 = 1 \;!$$

Ex: $r = 1 - 2\sin\theta$

Limaçon:



Ex: $r = \sin 2\theta$

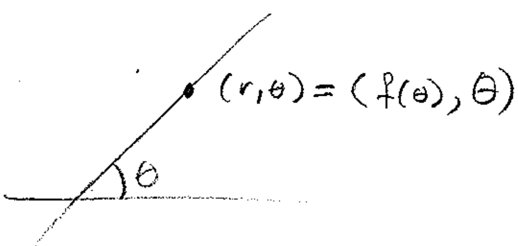


[See examples 4.9–4.12 in §9.4 for details]

Properties of curves in polar coordinates: (§9.5)

Suppose curve is given in polar coor by $r = f(\theta)$

Then rect coor are $(f(\theta)\cos\theta, f(\theta)\sin\theta)$, so



can also think of curve as given by

$$x(\theta) = f(\theta)\cos\theta$$
$$y(\theta) = f(\theta)\sin\theta$$

for a param. θ.

Can then compute tangent lines as before.

$$\text{slope} = \frac{y'(\theta)}{x'(\theta)} = \frac{f'(\theta)\cos\theta - f(\theta)\sin\theta}{f'(\theta)\sin\theta + f(\theta)\cos\theta}$$