Lecture 39: The thrilling conclusion

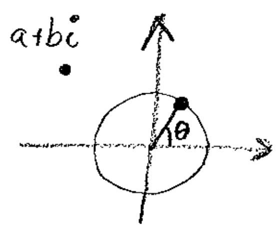
Reminder: HW#12 + Honors HW due Wed (separate sheets!)

Wed: Review; send topic suggestions to nmd@illinois.edu
[Enough chapter 9.]

Final: Friday 1:30–4:30 (here)

Extra Office Hours:
Thur: 1:30 – 4:00
Friday: 10–12

Last time: θ in $\mathbb{R}$: $e^{i\theta} = \cos\theta + i\sin\theta$



$a+bi = re^{i\theta}$ where (r,θ) are polar coordinates of (a,b).

$e^{i\pi} = -1$

Roots of complex numbers:

Ex: Find $\sqrt{1+i} = \sqrt{2}\,e^{i\pi/4}$ from last time.

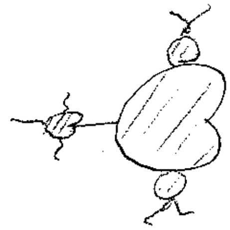
Want $z = re^{i\theta}$ with $z^2 = (re^{i\theta})^2 = r^2 \underbrace{e^{i2\theta}}_{|\,|=1} = 1+i$

So $r^2 = \sqrt{2} \Rightarrow r = \sqrt[4]{2}$ and $e^{i2\theta} = e^{i\pi/4}$

suggests $\theta = \pi/8$. So one square root of $1+i$ is

$\sqrt[4]{2}\,e^{i\pi/8}$; the other is $-\sqrt[4]{2}\,e^{i\pi/8} = \sqrt[4]{2}\,e^{i(9\pi/8)}$.

General Formula: if $\alpha = re^{i\theta}$ then its n^{th} roots are $\sqrt[n]{r}\, e^{i(\theta/n + \frac{2\pi k}{n})}$ for $k = 0, 1, \dots, n-1$

Mandelbrot Set:  [see Handout, last page.]

For $c \in \mathbb{C}$, consider the sequence: $c,\ c^2+c,\ (c^2+c)^2+c, \dots$

i.e. $z_0 = c$ and $z_n = z_{n-1}^2 + c$.

Ex: $c = 0$: $0, 0, 0, 0, 0, \dots \longrightarrow 0$

$c = 1$: $1, 2, 5, 26, 676, \dots \longrightarrow \infty$

$c = -\frac{1}{2} + \frac{1}{4}i$: $-\frac{1}{2} + \frac{1}{4}i,\ -\frac{5}{16},\ \frac{-103}{256} + \frac{i}{4},\ -\frac{26{,}255}{65{,}536} + \frac{25}{512}i \dots$

remains bounded

$$M = \left\{ c \text{ in } \mathbb{C} \;\middle|\; \text{the sequence } \{|z_n|\}_{n=0}^{\infty} \text{ is bounded} \right\}$$

Mathematics as a living subject

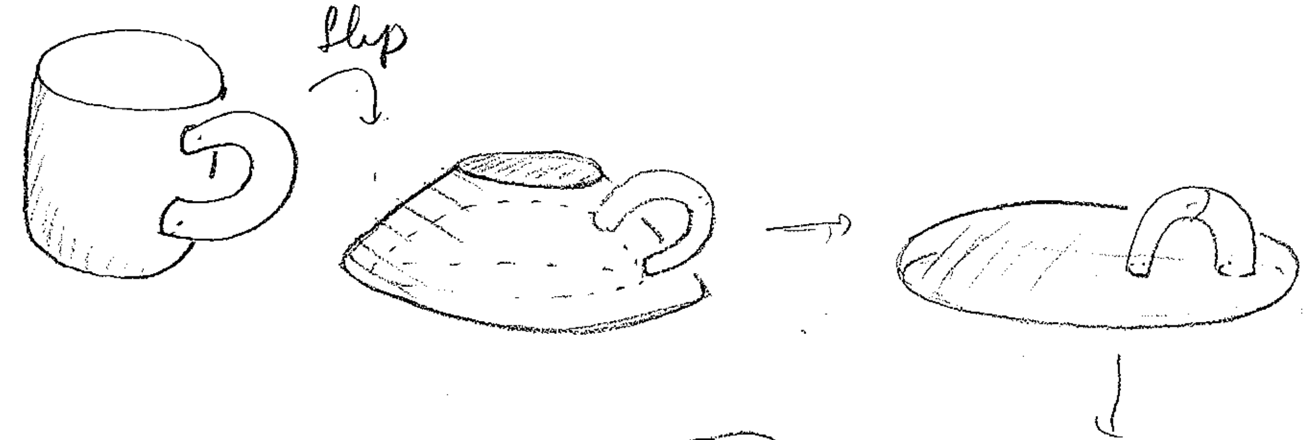
Most of this class is from 17^{th} - 18^{th} centuries

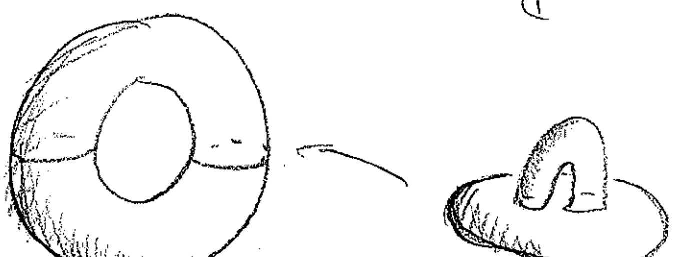
(Early 19^{th} century: Fourier series; formulation of ε-N def of limit mid 19^{th} century)

series using $e^{in\theta}$

In fact, new mathematics is discovered everyday. Much of it doesn't look like calculus

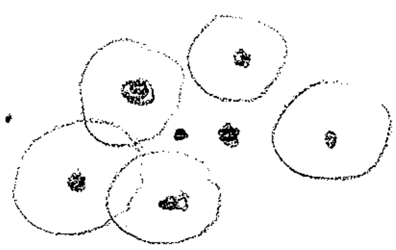
<u>Topologist</u>: someone who can't tell a coffee cup from a doughnut:





Surprisingly,
This has applications:

Robot arms, sensor networks...



Fundamental Theorem of Algebra is proved using topology. Fixed pt theorems and equilibria in economies.

The unreasonable effectiveness of mathematics

(# theory, cryptography, quantum computers.)

So consider

Math 347: Fundamental Mathematics

if you liked the honors HW!

Final: Comprehensive: Chapters 6, 8, 9, Handout   no Chapt 7.1

A little extra weight on parts of 9, handout

(e.g. was 1/6th of material, might be 1/5 of exam)

Course evaluations: