Lecture 40: Final Review

Final: Fri Dec 12: 1:30 – 4:30 in our usual room

Office Hours: Thurs 1:30 – 4:00
Fri 10 – 12

Can bring two sheets of notes

Review problems on course webpage

Course evaluations! | Talk about final (back of pg 90)

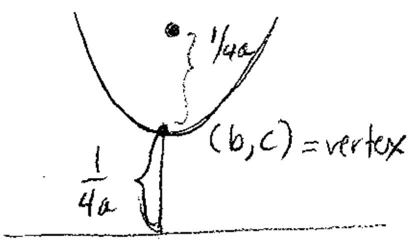
Complex Numbers: What is needed for the exam?
Ans: Same as any other part of the course.

Division: $\dfrac{3-i}{2+i} = (3-i)\left(\dfrac{1}{2+i}\right) = (3-i)\dfrac{2-i}{4+1} = 1-i$

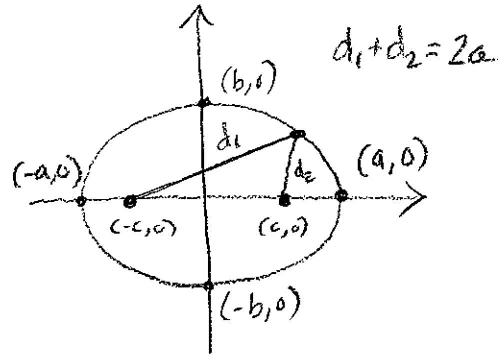
In general if $z = a+bi$ we defined $\bar{z} = a-bi$ and

$$\frac{1}{z} = \frac{1}{z}\frac{\bar{z}}{\bar{z}} = \frac{\bar{z}}{|z|^2} = \frac{a-bi}{a^2+b^2}$$

Conic Sections:

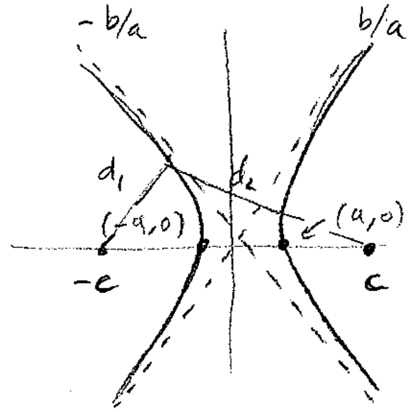


$$y = a(x-b)^2 + c$$



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$a > b \quad c = \sqrt{a^2-b^2}$



$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$C = \sqrt{a^2+b^2}$

$|d_1 - d_2| = 2a$

Note: Many things can be derived if you don't remember them. (e.g. find $d_1+d_2=2a$, $c=\sqrt{a^2-b^2}$ for the ellipse.)

Accuracy of Taylor polynomials:

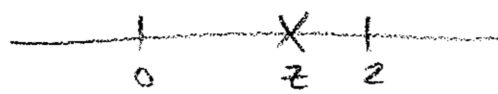
Estimate e^2 to within $\frac{1}{10}$ using $e^x = \sum_{k=0}^{\infty} \frac{1}{k!} x^k$

$P_n(x) = \sum_{k=0}^{n} \frac{1}{k!} x^k \leftarrow$ Taylor polynomial

Error: $R_n(2) = f(2) - P_n(2) = \frac{f^{(n+1)}(z)}{(n+1)!} 2^{n+1}$

where $f(x) = e^x$

and z is in $[0, 2]$.



So

$$|R_n(2)| = \frac{|e^z|}{(n+1)!} 2^{n+1} \leq \frac{|e^2|}{(n+1)!} 2^{n+1} \leq \frac{9}{(n+1)!} 2^{n+1}$$

as $|e^2| \leq 9$.

So if we take $n=7$, $|R_n(2)| \le \frac{9\, 2^8}{8!} = \frac{2}{35} < \frac{1}{10}$.

Thus

$$\sum_{k=0}^{7} \frac{1}{k!} 2^k = \frac{155}{21} \quad \text{is within } \frac{2}{35} \text{ of } e^2.$$

$$\approx 7.3809523\ldots$$

Testing convergence or divergence of series

E.g. $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k}{\sqrt{k}+\ln k}$, $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k \ln k}$, $\displaystyle\sum_{k=1}^{\infty} \frac{2^k}{k!}$.

From 1st midterm: Reduction formula for $\int \cos^n x\,dx$

$$\int \cos^n x\,dx = \int \cos^{n-1}x \left(\frac{d}{dx}\sin x\right) dx = \cos^{n-1}x \sin x + \int (n-1)\cos^{n-2}x \underbrace{\sin^2 x}_{1-\cos^2 x}\,dx$$

$$= \cos^{n-1}x \sin x + (n-1)\int \cos^{n-2}x\,dx - (n-1)\int \cos^n x\,dx$$

Thus $\displaystyle n\int \cos^n x\,dx = \cos^{n-1}x \sin x + (n-1)\int \cos^{n-2}x\,dx$

or $\displaystyle\int \cos^n x\,dx = \frac{1}{n}\cos^{n-1}x \sin x + \frac{(n-1)}{n}\int \cos^{n-2}x\,dx$

Final details:

Covers chapters 6, 8, 9, handout (not 7.1)

Extra weight on not covered in midterm III.

(was 1/6th of course, more like 1/5 of final.)

Have 3.5 as much time as a midterm,

but more like 2.5 as many problems.

Some may be more involved than before.

Still similar to HW.