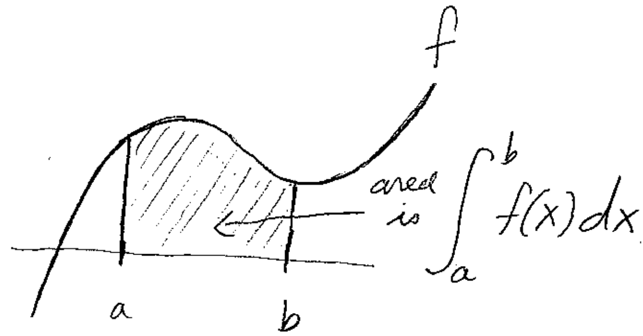
Math 231: Honors Calculus II : University of Illinois Fall 2008.

Lecture 1: [Distribute syllabus, survey]

Course Outline:

I. Methods of integration



You know:

$$\int x^2+1\,dx = \frac{1}{3}x^3 + x + C$$

and $$\int \frac{2x}{x^2+3}\,dx = \int \frac{1}{u}\,du = \ln|u| + C = \ln|x^2+3| + C$$

$$\boxed{\begin{aligned} u &= x^2+3 \\ du &= 2x \end{aligned}}$$

Will learn to integrate:

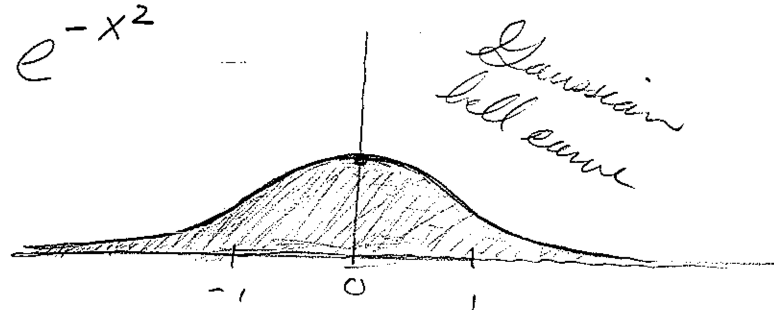
$$\int x \sin x\,dx \qquad \int \frac{1}{\sqrt{2+x^2}}\,dx \qquad \int \frac{1}{x^3+2x+1}\,dx \qquad \int x^7 e^x \sin x\,dx$$

$$\| $$

$$-x\cos x + \sin x + C$$

[Not everything can be integrated in "closed form", e.g. $\int \frac{1}{x+\sin x}\,dx$]

"Improper" integrals: $f(x) = e^{-x^2}$



$$\int_{-\infty}^{\infty} e^{-x^2}\,dx = \sqrt{\pi}$$

II: Sequences and series [Core of the course]

Sums: $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16} = \frac{15}{16}$

infinitely many terms

Infinite sums: $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\frac{1}{32}+\frac{1}{64}+\frac{1}{128}+\cdots = 1$

$$1+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+\frac{1}{25}+\cdots+\frac{1}{n^2}+\cdots = \frac{\pi^2}{6}$$

Not every such sum makes sense, though

$$1-10+100-1000+10000-100{,}000+\cdots\cdots = \text{nothing in particular}$$

and we will spend a lot of time figuring out when it does.

Approximation: [Motivation.]

$$\sin x = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 + \frac{1}{9!}x^9 - \cdots$$

query

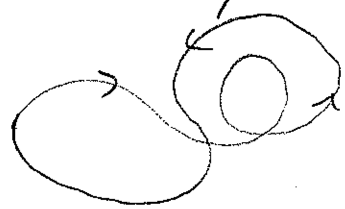
$$\sin 1 = 0.84147098\ldots$$

first 5 terms with $x = 1$ $= \frac{305353}{362880} = 0.84147100\ldots$

III. Other topics [Review of other classes.]

a) Complex numbers: making $x^2 = -1$ have a solution.

b) Curves in the plane, polar coordinates.



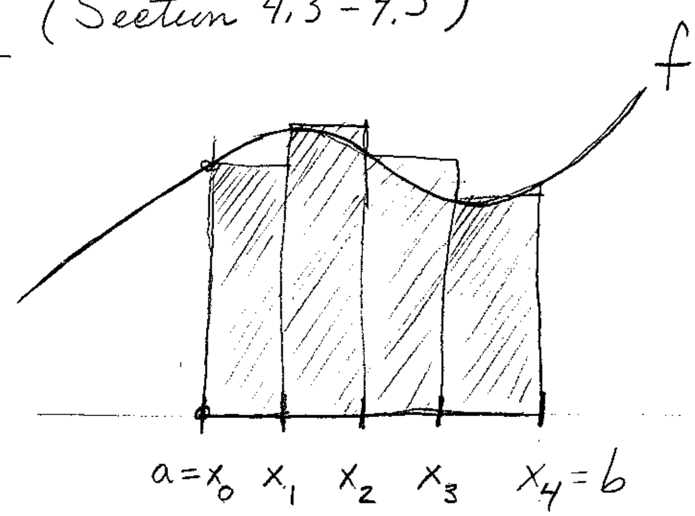
[c) Exponential growth (Differential Equations)
⋮ etc.]

[Discuss syllabus, point out HW due Friday]

Foundations of Integration (Section 4.3 - 4.5)

Approximating the definite integral

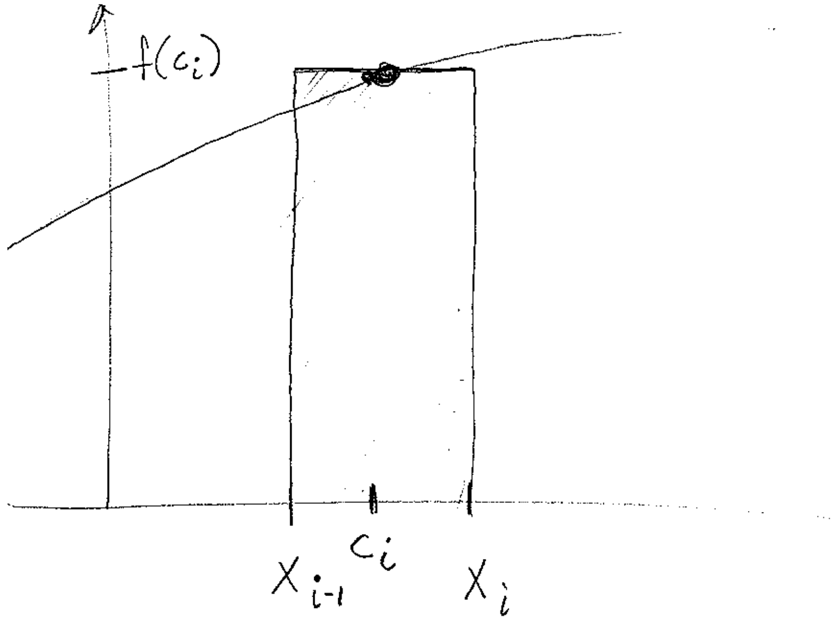
$$\int_a^b f(x)\,dx$$



Fix n and divide $[a, b]$ up into n equally sized pieces of length $\Delta x = \frac{b-a}{n}$

Now approximate the area with boxes as follows:

In each subinterval $[x_i, x_{i+1}]$ pick a point c_i and look at the box shown of height $f(c_i)$.



[In the first picture, I took $c_i = x_{i-1}$]

Then

$$\text{Area of all the boxes} = \sum_{i=1}^{n} \text{Area of } i^{th} \text{ box} = \underbrace{\sum_{i=1}^{n} f(c_i)\Delta x}_{\text{Riemann sum}}$$

<u>Def</u>: A function f on $[a,b]$ has integral L if

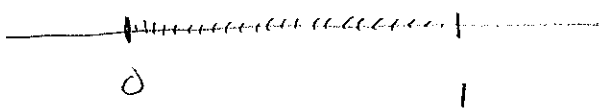
$$\lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\Delta x = L$$

for every choice of c_i. Say f is integrable on $[a,b]$ and write $\int_a^b f(x)\,dx = L$.

<u>Thm</u>: If f is continuous on $[a,b]$ then it is integrable on $[a,b]$.

Non integrable function: [First query class for ideas]

Define f on $[0,1]$ by $f(x) = \begin{cases} 1 & \text{if } x \text{ is rational, i.e. } x = p/q \text{ for integers } p + q \\ 0 & \text{otherwise} \end{cases}$



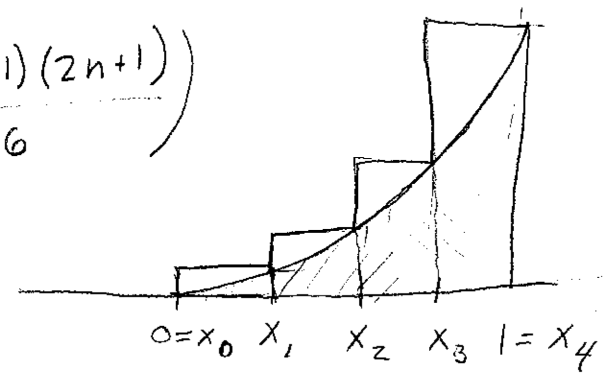
[Aside: Lebesgue integration signal processing, stocastic models, etc.]

Q: Who invented integration? A. Archimedes (286-211 BCE)

$$\text{Area} \approx \sum_{i=1}^{n} f(c_i)\,\Delta x = \sum_{i=1}^{n} \left(\frac{i}{n}\right)^2 \cdot \frac{1}{n}$$

$f(x) = x^2$

$$= \frac{1}{n^3} \sum_{i=1}^{n} i^2 = \frac{1}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)$$



As $n \to \infty$, we see

$$\int_0^1 x^2\,dx = 1/6$$

$x_i = \frac{1}{n} i$

$c_i = x_i$