Math 231 E1H: Honors Problem Set 3

Due date: In class on Wednesday, November 19.

1. Consider the function
$$f(x) = \begin{cases} e^{-1/x^2} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$
   (a) Sketch the graph of f.
   (b) Compute $f'(x)$. When x is 0, you'll need to go back to the definition of derivative in §2.2. You should find that $f'(0) = 0$.
   (c) Show that $f''(0) = 0$.

   In fact, $f^{(k)}(0) = 0$ for all k. Thus the Taylor series for f centered at 0 is just $\sum_{k=0}^{\infty} 0 = 0$; in particular, while it converges for all x, it only gives $f(x)$ for $x = 0$.
2. Find the Taylor series for $\sqrt{x}$ about a general center $c = a^2$.
3. Section 8.7: #43.
4. Section 8.7: #52.
5. Section 8.8: #47.