Math 231 E1H: Honors Problem Set 2

Due date: In class on Wednesday, October 22.

1. Consider the sequence $a_n = \frac{2n-1}{n}$ for $n \geq 1$.
   (a) Determine $\lim_{n\to\infty} a_n = L$.
   (b) Find an N so that
   $$|a_n - L| < 0.04 \quad \text{for all } n \geq N.$$
   Explain carefully why your choice of N works.
   (c) Find an N so that that
   $$|a_n - L| < 0.001 \quad \text{for all } n \geq N.$$
   Again, explain carefully why your choice of N works.
   (d) Directly from the definition on page 612, *prove* that $\lim_{n\to\infty} a_n = L$.
   Your answer should follow the template: "Let $\epsilon > 0$ be given. Choose N to be (something depending on ϵ). Then when $n \geq N$, we have
   $$|a_n - L| \leq (\text{ manipulations with all steps explained }) < \epsilon.$$
   Thus $\lim_{n\to\infty} a_n = L$."
2. Explain why
   $$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e$$
   Hint: Take natural logs of both sides and convert into something where you can apply L'Hopital's rule.
3. Given an example where $\sum_{k=1}^{\infty} a_k$ and $\sum_{k=1}^{\infty} b_k$ both diverge, but $\sum_{k=1}^{\infty}(a_k + b_k)$ converges.
4. Section 8.2: #44.
5. Section 8.3: #67 and #68.