Math 231 E1H: HW #12

Due date: In class on Wednesday, December 10.

Smith and Minton, Section 9.6: #13, 15, 17.
Hubbard and Hubbard (attached) Section 0.7: #2, 3, 6, 8, 11(a), 13.

Math 231 E1H: Honors Problem Set #4 (Corrected)

Due date: In class on Wednesday, December 10, **on separate sheets** from HW #12.

1. Recall that a function is *odd* if $f(-x) = -f(x)$ for all x, and *even* if $f(-x) = f(x)$ for all x. Suppose f and g are two functions and set $h(x) = f(x)g(x)$.
   (a) Suppose f is odd and g is even. What can you say about h?
   (b) Suppose f and g are both odd. What can you say about h?
   (c) Suppose f and g are both even. What can you say about h?

2. Suppose f is an odd function. Show that for each $L > 0$ one has
$$\int_{-L}^{L} f(x)\ dx = 0.$$
   Hint: break the integral into two pieces by splitting the interval $[-L, L]$ at 0. Then do a change of variables (u-substitution) to one of the new integrals to make it look more like the other one.

3. Suppose f is an even function. Find a relationship between
$$\int_{-L}^{L} f(x)dx \quad \text{and} \quad \int_{0}^{L} f(x)dx.$$
   Justify your answer carefully.

4. Use questions 1 and 2 to prove that if f is an odd function then its Fourier expansion has no cosine terms (i.e. $a_k = 0$ for $k > 1$). What, if anything, can you say about a_0?

5. Use the properties that $e^{a+b} = e^a e^b$ and $e^{i\theta} = \cos\theta + i\sin\theta$ for a real number θ to derive the sum formulas for sine and cosine.