Math 151a: HW #9 Due Wednesday, November 29

Reminder: The final will be given out on Wednesday, November 29, and due on Thursday, December 7.

1. From Hatcher: §2.2: #28, 36, 42. §2.3: #1.

2. Let H be a subgroup of the free group F_g of index k. Prove that H is free on $k(g-1)+1$ generators.

3. In an earlier problem, you dealt with gluings of triangles. This question will deal with the 3-dimensional case. Consider a finite collection of 3-simplices $T_1, T_2, \dots, T_n$. Create a space X by gluing the faces of the T_i in pairs. In particular, every face of T_i is glued to precisely one face of some T_j. Prove that X is a 3-manifold if and only if $\chi(X) = 0$. Thus not every such gluing gives a 3-manifold, and indeed most don't.

   Notes: You will need to use the classification of compact surfaces in your proof. Do *not* assume the fact mentioned in class that odd dimensional manifolds have Euler characteristic 0.