

## Math 151a: HW #9 Due Wednesday December 1

**Reminder:** The final will be given out on Wednesday, December 1, and due on Wednesday, December 8.

1. From Hatcher: §2.2: #28, 36, 42. §2.3: #1.
2. Let  $H$  be a subgroup of the free group  $F_g$  of index  $k$ . Prove that  $H$  is free on  $k(g - 1) + 1$  generators.
3. In an earlier problem, you dealt with gluings of triangles. This question will deal with the 3-dimensional case. Consider a finite collection of 3-simplices  $T_1, T_2, \dots, T_n$ . Create a space  $X$  by gluing the faces of the  $T_i$  in pairs. In particular, every face of  $T_i$  is glued to precisely one face of some  $T_j$ . Prove that  $X$  is a 3-manifold if and only if  $\chi(X) = 0$ . Thus not every such gluing gives a 3-manifold, and indeed most don't.

**Notes:** You will need to use the classification of compact surfaces in your proof. Do *not* assume the fact mentioned in class that odd dimensional manifolds have Euler characteristic 0.